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The blast waves from asymmetrical explosions

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From Whitham's ray-shock theory and the Brinkley–Kirkwood theory of shock propagation, a general theory for the propagation of asymmetrical blast waves of arbitrary shapes and strengths is developed in this paper. The general theory requires the simultaneous numerical solution of a set of partial differential equations and a pair of ordinary differential equations. If the shock shape is assumed to be known and remains invariant with time then the geometrical and the dynamical relationships in the theory can be decoupled. In this case the solution simply requires the integration of the ordinary differential equations governing the dynamics of the blast motion since the geometry is already known. As a specific example the asymmetrical blast waves generated by the rupture of a pressurized ellipsoid are studied. The peak pressure is calculated by assuming that the shock surface remains ellipsoidal for all times and that the peak overpressure decay rate of the blast depends on the local curvature. For weak shocks, it is found that the degree of directionality is more pronounced than for stronger shocks. For weak blasts the present theory agrees with the solution based on acoustic theory. Experimental results on the shock trajectories for asymmetrical blast waves generated by exploding wires are found to agree with the present theory.

1. Introduction

It is well established that the blast wave from accidental chemical explosions is in general far from ideal. This is due to the fact that the energy release is fairly distributed temporally and spatially, and wave symmetry is the exception rather than the rule. Thus the conventional practice of damage assessment or risk evaluation based on the estimate of a TNT equivalent cannot yield realistic results. This fact has been recognized universally, and the study of non-ideal blast waves has received considerable attention in recent years. A summary of these studies can be found in the recent review article by Strehlow & Baker (1975). In most of the existing work, attention has been focused on the non-idealisms arising from the finite rate of energy deposition. The question of wave asymmetry has not been fully explored. This is an important aspect because the damage patterns of most accidental explosions all demonstrate a high degree of directionality. The present paper is concerned with the propagation of asymmetrical blast waves generated by non-spherical energy sources of finite size.

The formalisms developed in previous studies of asymmetrical blast waves do not lend themselves readily to the description of the types of situation in accidental chemical explosions. In general the blast wave generated in an accidental explosion is of moderate strength and the degree of wave asymmetry is quite large. The previous studies mostly dealt with very strong blast waves (i.e. $M_s \rightarrow \infty$) with $\gamma \rightarrow 1$ where the 'snow-plough' model is applicable (Laumbach & Probstein 1969) or with blasts with small deviations from sphericity in which asymmetry can be considered as a small perturbation to the motion of the spherical wave (Panarella & Savic 1968). Thus there arises the need for an appropriate theory for the highly asymmetrical blast wave of moderate strength that is generally associated with chemical explosions.

The theory developed in this paper is based on Whitham's (1957*a*, 1959) 'ray-shock' theory. The 'ray-shock' theory uses the instantaneous shock shapes and the orthogonal trajectories (i.e. rays) from the shock surface as a set of curvilinear coordinates. Then from purely kinematic considerations, an equation (much like the eikonal equation in geometrical optics) for the ray direction as a function of the ray-tube area A and the shock Mach number M_s can be derived. To complete the formalism, an independent relationship for the shock Mach number and the ray-tube area (i.e. $A(M_s)$) must be specified. Whitham used the so-called 'Chester function', which was derived originally by Chester (1954) and later by Chisnell (1957) and Whitham (1957*b*) on a different basis. The essence of Whitham's theory is to treat the propagation of an arbitrarily shaped shock wave as that of a shock wave in a tube of variable area, the area variation being a function of the shock motion itself. Whitham's theory is quite successful for shock diffraction problems, but cannot be applied directly to asymmetrical blast propagation. This is due to the inapplicability of the 'Chester function' for the description of blast decay. The Chester function relates the shock Mach number to the local area. Thus the mechanism for the decay of the shock is considered to be primarily due to area increase. For blast waves, the decay is due to entirely different mechanisms: the non-uniform flow structure behind the shock. Thus a planar blast wave decays even though there is no area variation. For Whitham's ray-shock theory to be applicable to asymmetrical blasts, it is clear that an appropriate expression which contains the blast decay mechanisms must be used in place of the Chester function. In the present formalism, this expression is developed from the Brinkley–Kirkwood (1947) theory. Thus in essence the present theory is simply Whitham's original ray-shock theory but with the Chester function replaced by the appropriate expression developed from the Brinkley–Kirkwood theory of shock propagation so as to incorporate the correct blast decay mechanisms.

Whitham's theory is an approximate theory and represents considerable simplification when compared with the solution obtained via direct numerical integration of the unsteady three-dimensional conservation equations of gasdynamics. However, for arbitrary shock shapes, the solution using Whitham's theory is by no means trivial and involves the solution of a partial differential equation of the hyperbolic type together with a set of ordinary differential equations from Brinkley & Kirkwood's theory for the shock decay. Thus, in the present paper we shall restrict ourselves to shock shapes with rotational symmetry. By making the further assumption that the shock retains its shape at all times, the problem is considerably simplified in that, if only the shock motions along the axis of rotational symmetry and perpendicular to this axis are desired, the solution can readily be obtained from the simultaneous integration of six ordinary differential equations. The particular shock shape studied in this paper is the ellipsoid, and we shall consider the blast to be generated initially by the sudden rupture of a pressurized ellipsoidal container. The acoustic solution is also developed in this paper. For weak asymmetric blast waves, it is found to agree extremely well with the more general theory developed from the Brinkley–Kirkwood

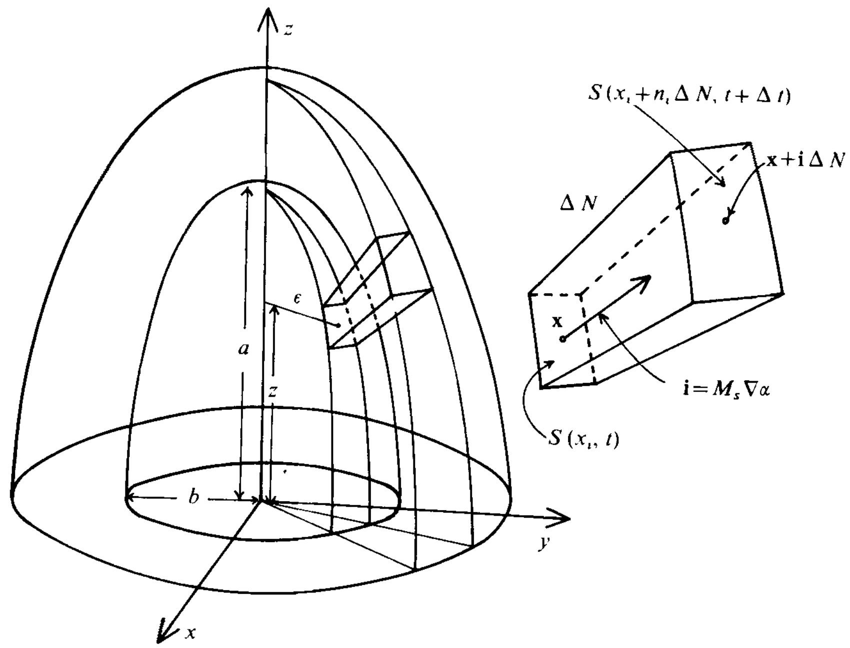


FIGURE 1. A sketch of a ray tube of an asymmetric shock wave. $S(x_i, t)$ and $S(x_i+n_i\Delta N, t+\Delta t)$ are the shock surfaces at times t and $t+\Delta t$. $\mathbf{i}$ is a unit vector normal to the shock surface. $\epsilon^2 = x^2+y^2$.

theory and Whitham's ray-shock theory. The present theory also agrees with experimental results for asymmetrical blast waves generated by an exploding wire 2 cm long.

2. General theory

We shall briefly rederive the working relationships of Whitham's ray-shock theory. Let the shock shape be denoted by the equation $S(x_i t) = 0$ (see figure 1). By differentiating S with respect to t, we obtain

$$\partial S/\partial t + \nabla S . d\mathbf{x}/dt = 0. \tag{2.1}$$

The magnitude of the shock velocity (i.e. the velocity normal to the shock surface) is then given by

$$\dot{R}_s = \left|\frac{d\mathbf{x}}{dt}\right| = -\frac{1}{|\nabla S|}\frac{\partial S}{\partial t}. \tag{2.2}$$

Following Whitham, we write the shock surface as

$$S(x_i t) = c_0 t - \alpha(x_i) = 0.$$

Hence (2.2) becomes

$$M_s = \dot{R}_s/c_0 = 1/|\nabla\alpha|, \tag{2.3}$$

where M_s is the shock Mach number.

Defining a 'ray' as a trajectory perpendicular to the surface, we may write

$$\mathbf{i} = \nabla\alpha/|\nabla\alpha| = M_s\nabla\alpha, \tag{2.4}$$

where $\mathbf{i}$ is the unit vector in the ray direction at any point. By considering a narrow ray tube whose end sections are part of two successive shock surfaces $\alpha(x_i)$, it can be shown (Whitham 1957*a*, 1959) that

$$\nabla.(\mathbf{i}/A) = \nabla.(M_s \nabla\alpha/A) = 0, \tag{2.5}$$

where A is proportional to the cross-sectional area of the ray tube (measured as the area of the surface $\alpha(x_i)$ = constant inside the tube at that section). In most cases A can be taken to represent the area of the ray tube directly. Equations (2.3) and (2.5) yield a pair of relationships for the three quantities M_s, $\alpha(x_i)$ and A. If a third independent expression for the three quantities can be found, the formulation will then be complete.

In the original work of Whitham the 'Chester function', which links the ray-tube area A with the shock Mach number M_s, is used. With the relation between the area and Mach number specified, the motion of a shock wave of given initial shape and Mach number distiibution can be found using the two geometrical conditions (2.3) and (2.5). Examples of the use of the ray-shock theory can be found in the original papers of Whitham (1957*a*, 1959).

As mentioned previously, we need to seek a more appropriate expression for the dependence of the shock strength M_s on the local ray-tube area A for blast waves. In a previous paper (Bach, Chiu & Lee 1975) the Brinkley–Kirkwood theory has been shown to provide a good description, particularly for moderate-strength and weak blast waves. However, the formulation given was based on spherical or cylindrical symmetry. In terms of the non-dimensional overpressure Z ($= \Delta P_s/P_0$), the propagation of spherical blast waves was found to be described by the following differential equations (see equations 16 and 17 of Bach *et al.* 1975):

$$\frac{dZ}{dR} = -F_1(Z)\left[F_2(Z)\frac{2}{R} + P_0\nu(Z)\frac{4\pi R^2}{W(R)}\right], \tag{2.6}$$

$$\frac{dW}{dR} = -\frac{4\pi R^2}{\gamma-1}P_0F_3(Z), \tag{2.7}$$

where
$$F_1(Z) = \frac{2Z^4(\gamma+1)/\gamma}{16\gamma^2+4Z\gamma(5\gamma+1)+Z^2(\gamma+1)(5\gamma-1)}, \tag{2.8a}$$

$$F_2(Z) = \frac{\gamma}{(\gamma+1)Z^3}[4\gamma^2+2\gamma(3\gamma-1)Z+(2\gamma^2-\gamma+1)Z^2], \tag{2.8b}$$

$$F_3(Z) = (1+Z)^{1/\gamma}\frac{2\gamma+(\gamma-1)Z}{2\gamma+(\gamma+1)Z}-1, \tag{2.8c}$$

$$\nu(Z) = 1-\tfrac{1}{3}e^{-Z}. \tag{2.8d}$$

Here the Brinkley–Kirkwood (B–K) energy integral $W(R)$ is defined by Brinkley (1972) to be the work done by the pressure along the particle path $r = r(R,t)$, i.e.

$$W(R) = \int_{t(R)}^{\infty} Pu4\pi r^2(R,t)\,dt. \tag{2.9}$$

Equations (2.6) and (2.7) provide a pair of ordinary differential equations for the propagation of spherical shock waves. If the initial conditions $(Z(R_i),\ W(R_i))$ or equivalently $(Z(R_i),\ [dZ/dR]_{R_i})$ are known, the subsequent motion of the shock can be obtained by integrating (2.6) and (2.7). It is to be noted that in the Chester function only the initial shock strength M_s (or Z) need be specified. In the B–K theory the initial rate of decay (i.e. $[dZ/dR]_{R_i}$) or equivalently the shock wave energy $W(R_i)$ must be specified in addition to the initial shock strength Z. This is due to the fact that in the B–K theory the attenuation of the shock wave is due not to the area change alone, but to the expansion gradient behind the shock as well. Thus the initial rate of decay dZ/dR of the shock wave must also be specified. For a more detailed discussion of the B–K theory and the derivation of the required relationships, the recent paper of Bach *et al.* (1975) should be consulted.

Thus far, the B–K theory has been formulated only on the basis of spherical, cylindrical or planar symmetry. In order that the results be valid for an arbitrary shock tube with given cross-sectional area $A(R)$, (2.6) and (2.7) have to be modified. Careful examination of the B–K formalism in an arbitrary shock tube shows that all we have to do is (i) to replace $4\pi R^2$ by $A(R)$, (ii) to replace $2/R$ by $A^{-1}dA/dR$ and (iii) to introduce

$$L \equiv W(R)/P_0 A(R). \tag{2.10}$$

With these changes (2.6) and (2.7) become

$$\frac{dZ}{dR} = -F_1(Z)\left[F_2(Z)\frac{1}{A}\frac{dA}{dR}+\frac{\nu(Z)}{L}\right], \tag{2.11}$$

$$\frac{dL}{dR} = -\frac{F_3(Z)}{\gamma-1}-\frac{L}{A}\frac{dA}{dR}. \tag{2.12}$$

It is important to note that for asymmetrical shock waves L is a well-defined quantity at any point but W is not. Hence the transformation from W to L in developing a theory of asymmetrical blast waves is essential.

The formulation of the present theory is now complete. The two geometrical relationships (2.3) and (2.5) are identical to those in Whitham's original theory. However, for the dynamical condition, the Chester function is now replaced by the two ordinary differential equations (2.11) and (2.12). With the initial shock shape and shock strength distribution as well as the initial rate of decay of the shock (or equivalently the shock wave energy) specified, (2.3), (2.5), (2.11) and (2.12) must be solved simultaneously to achieve the description of the subsequent motion of the blast wave.

3. Ellipsoidal blast waves

In the general theory given in the previous section, the motion of the blast wave is described by the pair of ordinary differential equations derived from the Brinkley–Kirkwood theory. However, in these equations the area variation in the direction of motion (i.e. dA/dR) must be known. It is the geometrical relationships from Whitham's ray-shock theory that give this area variation as a function of the shock strength M_s. Thus the geometrical and the dynamical conditions are coupled through this area variation (or the shock shape) and thus (2.3), (2.5), (2.11) and (2.12) must be solved simultaneously. This is not an easy task in general, for the geometrical conditions lead to partial differential equations of the hyperbolic type. The solution of these equations must be achieved numerically using, for example, the method of characteristics, as in the solution of the ordinary conservation equations in compressible flow. However, if we could decouple the geometrical from the dynamical conditions, then we should simply have a set of ordinary differential equations to integrate for the motion of the shock wave. Decoupling requires the area variation along the ray tube to be specified *a priori*. In other words, if the shape of the blast wave can be specified then the solution of the problem is greatly simplified.

From experimental observation we note that asymmetrical blasts with rotational symmetry tend to retain their shapes with time. In other words, if the initial shape of the shock wave is ellipsoidal, then its shapes at later times will not deviate much from an ellipsoid. We shall consider in this paper the motion of an ellipsoidal blast wave under the assumption that the shape remains ellipsoidal at all times. This is not too severe a restriction since the ratio of the major to the minor axis (i.e. a/b) is allowed to change with time according to their decay rate. Thus the degree of ellipticity can vary with time, and the ratio a/b approaching unity in the asymptotic limit of a spherical wave is in accord with experimental observations.

For an ellipsoidal shock surface, we have (see figure 1)

$$z^2/a^2 + \epsilon^2/b^2 = 1, \tag{3.1}$$

where z and ϵ are the components along the z axis (the axis of rotational symmetry) and in the x, y plane respectively. r has already been used to denote direction along a ray tube. In what follows, we shall use 'major axis' to denote the z axis and 'minor axis' to denote the x, y plane. From (2.5), we can write

$$\nabla . \frac{\mathbf{i}}{A} = -\frac{\mathbf{i}}{A^2} . \nabla A + \frac{1}{A} \nabla . \mathbf{i} = 0,$$

and since ∇A and $\mathbf{i}$ are both normal to the shock surface and r is in the direction of a ray, $|\nabla \mathbf{A}| \equiv dA/dr$ and the above equation gives

$$A^{-1} dA/dr = \nabla . \mathbf{i}. \tag{3.2}$$

From (3.1), the unit vector $\mathbf{i}$ can be written as

$$\mathbf{i} = \left(\frac{\epsilon}{b^2}\mathbf{e}_\epsilon + \frac{z}{a^2}\mathbf{e}_z\right)\left[\left(\frac{\epsilon}{b^2}\right)^2 + \left(\frac{z}{a^2}\right)^2\right]^{-\frac{1}{2}} \tag{3.3}$$

and if the above is used to evaluate the divergence of $\mathbf{i}$ (3.2) becomes

$$\frac{1}{A}\frac{dA}{dr} = \left(\frac{\epsilon^2}{b^4} + \frac{z^2}{a^4}\right)^{-\frac{3}{2}} \left[\frac{\epsilon^2}{b^4}\left(\frac{1}{a^2} + \frac{1}{b^2}\right) + \frac{2z^2}{a^4 b^2}\right]. \tag{3.4}$$

By substituting the above into (2.11) and (2.12), the motion of the shock at any given location on the surface of the ellipsoid (ϵ, z) can be obtained. Of particular interest is the motion along the major axis $\epsilon = 0$, $z = a$ and along the minor axis $\epsilon = b$, $z = 0$. Specializing (3.4) to these two locations, we get

$$A^{-1} dA/da = 2a/b^2, \tag{3.5}$$

$$A^{-1} dA/db = 1/b + b/a^2. \tag{3.6}$$

If the above are substituted into (2.11) and (2.12) the resultant set of ordinary differential equations is

$$da/dt = M_{sa} c_0, \tag{3.7}$$

$$\frac{dZ_a}{da} = -F_1(Z_a)\left[\frac{2a}{b^2}F_2(Z_a) + \frac{\nu(Z_a)}{L_a}\right], \tag{3.8}$$

$$\frac{dL_a}{da} = -\frac{F_3(Z_a)}{\gamma-1} - \frac{2a}{b^2}L_a, \tag{3.9}$$

$$db/dt = M_{sb}c_0, \tag{3.10}$$

$$\frac{dZ_b}{db} = -F_1(Z_b)\left[\left(\frac{1}{b}+\frac{b}{a^2}\right)F_2(Z_b) + \frac{\nu(Z_b)}{L_b}\right], \tag{3.11}$$

$$\frac{dL_b}{db} = -\frac{F_3(Z_b)}{\gamma-1} - \left(\frac{1}{b}+\frac{b}{a^2}\right)L_b, \tag{3.12}$$

where
$$M_s^2 = 1 + \frac{\gamma+1}{2\gamma}Z.$$

The subscripts a and b in the above equations denote the conditions along the major (z) axis and the minor (x, y plane) axis, respectively. Once the initial conditions (Z_a, Z_b, L_a, L_b) at $t = t_i$, where $a = a_i$ and $b = b_i$, have been specified, the subsequent propagation of the blast wave can be obtained by integrating the set of equations (3.7)–(3.12) simultaneously.

We shall consider the blast wave to be generated by the rupture of a pressurized ellipsoid with initial overpressure ΔP_i. Following Brode's (1955) definition, the energy of the ellipsoid is written as

$$W_i = \tfrac{4}{3}\pi a_i b_i^2 \Delta P_i/(\gamma - 1), \tag{3.13}$$

where a_i and b_i denote the major and minor axes of the pressurized ellipsoid prior to rupture. For the initial conditions, we assume that

$$L_a = L_b = W_i/P_0 A, \tag{3.14}$$

where A is the surface area of the ellipsoid. Since $R_0 = (W_i/P_0)^{\frac{1}{3}}$ is defined to be the explosion length, L_a and L_b are both equal to R_0^3/A, which obviously has the dimensions of length. The initial shock strength Z can be found from the one-dimensional shock-tube relationship for any given value of the overpressure ΔP_i. Again we shall assume that $M_{sa} = M_{sb}$, or in other words, that the initial shock strength distribution is uniform for the ellipsoid.

It should be noted that an alternative method can be used to initiate the integration of (3.7)–(3.12). For a given ΔP_i, the initial shock strength can be obtained from one-dimensional shock-tube theory as described previously. However, the initial rate of decay of the shock wave can be obtained by performing a perturbation analysis of the one-dimensional shock-tube flow to account for the geometrical effects. In this manner the initial rate of decay of the shock wave will not be identical along the major and the minor axes. Thus it appears that this method of determining the initial conditions is more exact. However, it was shown in our earlier studies on the rupture of pressurized spheres, where both methods of solution were tried, that the former method of using the energy to start the numerical integration is superior. The nature of the B–K theory is such that the solution is not too sensitive to the value of the energy. Thus the assumption that $L_a = L_b$ initially is expected to be quite good. Also, if the initial rate of decay is specified, extreme accuracy must be used, otherwise the integration is unstable.

The results obtained using both methods for the initial conditions agree with each other for the spherical cases analysed previously. Hence, in the present work the initial energy is used to start the numerical integration of (3.7)–(3.12).

4. Limiting solution for weak explosions

If the overpressure ratio $\Delta P_i/P_0 \ll 1$, then the blast wave generated upon explosion of the ellipsoid (or any arbitrary convex pressurized vessel) can be described by acoustic theory. The restriction to convex vessels is made to avoid the complexity of caustic formation or focusing of the wave front. Perhaps the simplest approach is first to construct the solution from a known solution using the principle of supposition. Nonlinear effects are accounted for at the end by using Whitham's (1956) weak-shock theory or on the basis of Gottlieb's solution (Gottlieb & Glass 1974; Gottlieb 1974). Specifically, the acoustic N-wave radiation from the rupture of a pressurized sphere is well known. Thus for an arbitrarily shaped pressurized vessel, we can consider the vessel to be made up of a large number of elementary pressurized spheres. The pressure field upon rupture of the vessel can then be obtained by summing the individual contributions from the elementary spheres.

The overpressure field at time t at the origin $r = 0$ due to the rupture at $t = 0$ of a pressurized sphere of overpressure ΔP_i and radius R_i situated at r can be written as (see figure 2a)

$$\Delta P(0,t) = (\Delta P_i/2r)(r-c_0t)S, \tag{4.1}$$

where $S = 1$ for $|r-c_0t| < R_i$ and $S = 0$ otherwise (Landau & Lifshitz 1966, p. 267). For a volume dv of an arbitrarily shaped pressurized vessel with overpressure ΔP_i, the number of elementary pressurized spheres of radius R_i will be $dv/(\frac{4}{3}\pi R_i^3)$. Thus the pressure field at $r = 0$ due to the rupture of such a vessel is given by the following integral:

$$\Delta P(0,t) = \lim_{R_i\to 0}\frac{3}{4\pi R_i^3}\int dv\,\frac{\Delta P_i}{2r}(r-c_0t)S. \tag{4.2}$$

The volume dv can be written as $A(r)\,dr$, and we note that, at any instant of time t, only a thin slice (thickness $2R_i$) of the pressurized vessel contributes to the pressure field at the origin $r = 0$ (figure 2b). By expanding $A(r)$ in a Taylor series about $r = c_0t$ and by retaining only the first-order term in that series, (4.2) becomes simply

$$\Delta P(0,t) = \frac{\Delta P_i}{4\pi c_0 t}\frac{dA}{dr}\bigg|_{r=c_0t}. \tag{4.3}$$

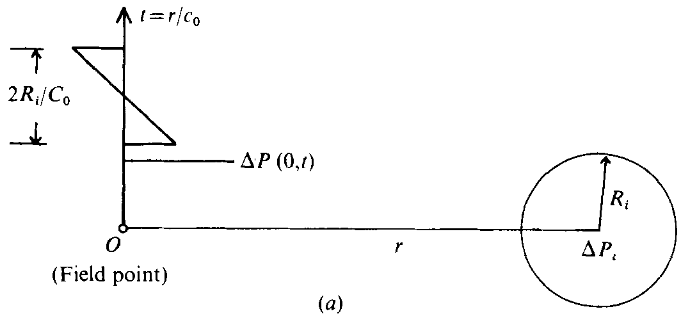
Obviously, $[dA/dr]_{r=c_0t}$ depends on the shape of the exploding vessel itself. The peak overpressure can easily be obtained from (4.3) by taking r to be the shortest distance from the field point to the vessel. For a spherical vessel of radius R, whose centre is at r_0+R, the function $A(r)$ is given by

$$A(r) = (2\pi r_0 R/r)(r-r_0)$$

for $r-r_0 \ll r_0$. Substituting the above into (4.3) yields

$$\Delta P(0,t = r_0/c_0) = \Delta P_i R/2(r_0+R), \tag{4.4}$$

which is exactly the peak of the N-wave at a field point r_0+R from the explosion of a pressurized sphere of radius R and overpressure ΔP_i.



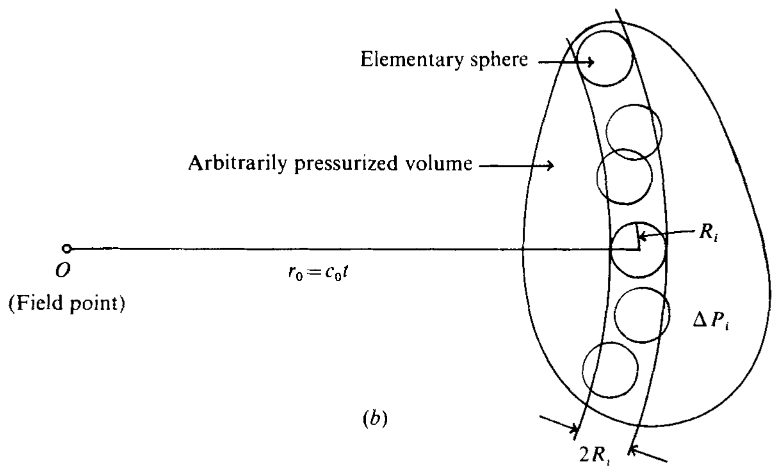


FIGURE 2. (*a*) The overpressure N-wave due to the rupture of a pressurized sphere. (*b*) The overpressure waves of each elementary sphere (radius R_i) centred in the thin slice contributing towards the overpressure wave at the field point at time t.

We now apply (4.3) to evaluate the peak overpressure along the major and minor axes of an exploding pressurized ellipsoid. Along the major axis, we note that the end of the ellipsoid is a spherical cap with radius $R = b_i^2/a_i$, where a_i and b_i are the major and minor axes of the ellipsoid, respectively. Thus (4.4) gives the peak overpressure along the major (z) axis as

$$\Delta P(z = a) = \frac{\Delta P_i}{2} \frac{b_i^2/a_i}{a - a_i + b_i^2/a_i}, \tag{4.5}$$

where a is the distance of the field point from the centre of the ellipsoid. Similarly, the peak overpressure along the minor axis (x, y plane) is found to be

$$\Delta P(\epsilon = b) = \frac{\Delta P_i}{2} \left(\frac{a_i^2/b}{b - b_i + a_i^2/b_i} \right)^{\frac{1}{2}}, \tag{4.6}$$

where b is the distance of the field point from the centre of the ellipsoid. From (4.5) and (4.6) we note that for $a_i = b_i = R$, when the ellipsoid becomes a sphere, (4.5) and (4.6) both reduce to the equation (4.4) for the spherical case. The peak overpressure of the wave at the instant of rupture of the sphere or ellipsoid is found to be $\frac{1}{2}\Delta P_i$, which corresponds to the value obtained in the limit of very small overpressure ratio from the one-dimensional shock-tube formula. In the far field, where a or b is large compared with a_i and b_i, (4.5) and (4.6) yield

$$\Delta P(z=a) = \frac{\Delta P_i}{2a}\frac{b_i^2}{a_i}, \quad \Delta P(\epsilon = b) = \frac{\Delta P_i}{2b}a_i. \tag{4.7}$$

Even when $a = b$, the ratio of these overpressures does not approach unity; instead, it approaches $(b_i/a_i)^2$ as $a = b \to \infty$. Thus we see that the initial degree of asymmetry is retained even in the far field although the wave shape approaches sphericity (i.e. $(a_i + c_0 t)/(b_i + c_0 t) \to 1$ as $t \to \infty$). From (4.6), we note that for $a_i \to \infty$ (b_i being finite)

$$\Delta P(\epsilon = b) \simeq \tfrac{1}{2}\Delta P_i (b_i/b)^{\frac{1}{2}}, \tag{4.8}$$

i.e. we recover the result for cylindrical waves, where the peak decays like $b^{-\frac{1}{2}}$. From (4.5), we note that for $b_i \to \infty$ (a_i being finite)

$$\Delta P(z=a) \simeq \tfrac{1}{2}\Delta P_i, \tag{4.9}$$

which is the planar-wave result, in which the wave amplitude remains invariant. Thus the solution given by (4.5) and (4.6) yields the appropriate results in the various limiting cases.

It should be pointed out that (4.5) and (4.6) are based on linear acoustic theory, in which superposition holds. These expressions are not accurate in the far field, when wave distortion can no longer be ignored. In this paper, we shall use the technique of Landau (1948), Whitham (1956) and Gottlieb (1974) to account for nonlinear effects in the far field. According to Gottlieb the peak overpressure from the explosion of a pressurized sphere of overpressure ΔP_i and radius R_i is given by

$$\Delta P(r) = \frac{\Delta P_i R_i}{2r}\left(1 + \frac{\gamma+1}{4\gamma}\frac{\Delta P_i}{P_0}\ln\frac{r}{R_i}\right)^{-\frac{1}{2}}. \tag{4.10}$$

We recall that in the linear acoustic theory R_i is equal to the length of the positive phase of the pressure wave while r is the radius of curvature of the shock surface. Equation (4.10) implies that the decay law is completely determined by local properties of the pressure pulse like the peak overpressure, the pulse length and the curvature of the shock surface. Hence extension from a spherical explosion to an asymmetrical explosion is trivial. Along the major axis, the pulse length is closed to a_i. Hence the modified overpressure should be

$$\Delta P(z=a) = \frac{\Delta P_i}{2}\frac{b_i^2/a_i}{a - a_i + b_i^2/a_i}\left(1 + \frac{\gamma+1}{4\gamma}\frac{\Delta P_i}{P_0}\ln\frac{a}{a_i}\right)^{-\frac{1}{2}}. \tag{4.11}$$

Along the minor axis we have

$$\Delta P(\epsilon = b) = \frac{\Delta P_i}{2}\left(\frac{a_i^2/b}{b - b_i + a_i^2/b_i}\right)^{\frac{1}{2}}\left(1 + \frac{\gamma+1}{4\gamma}\frac{\Delta P_i}{P_0}\ln\frac{b}{b_i}\right)^{-\frac{1}{2}}. \tag{4.12}$$

5. Results and discussion

All numerical results are based on a perfect gas with $\gamma = 1{\cdot}4$. The shock overpressure ratio $\Delta P_s/P_0$ is plotted *vs.* shock position r/R_0 ($r = a$ for the major axis and $r = b$ for the minor axis in the x, y plane) for various initial overpressures ΔP_i of the ellipsoid for two typical ellipticities $a_i/b_i = 2$ and $0{\cdot}5$, in figures 3 (a) and (b) respectively.

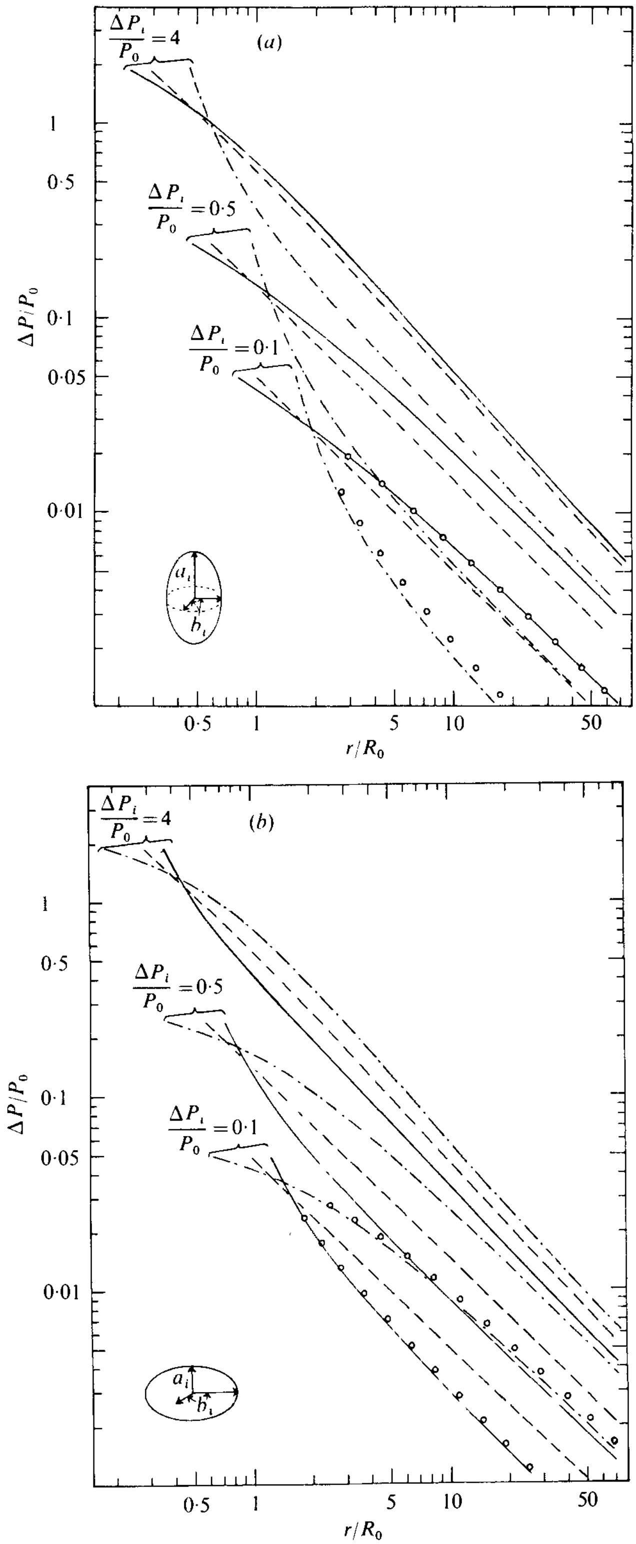


FIGURE 3. Shock overpressure $\Delta P/P_0$ *vs.* distance r/R_0 for ellipsoidal shock waves due to the rupture of (*a*) a prolate pressurized ellipsoid with $a_i/b_i = 2$ and (*b*) an oblate pressurized ellipsoid with $a_i/b_i = 0.5$. The spherical result is also shown for comparison. The initial pressure ratio P/P_0 takes the values 5, 1·5 and 1·1. —·—, major axis; ——, minor axis; ---, spherical; ○, acoustic theory.

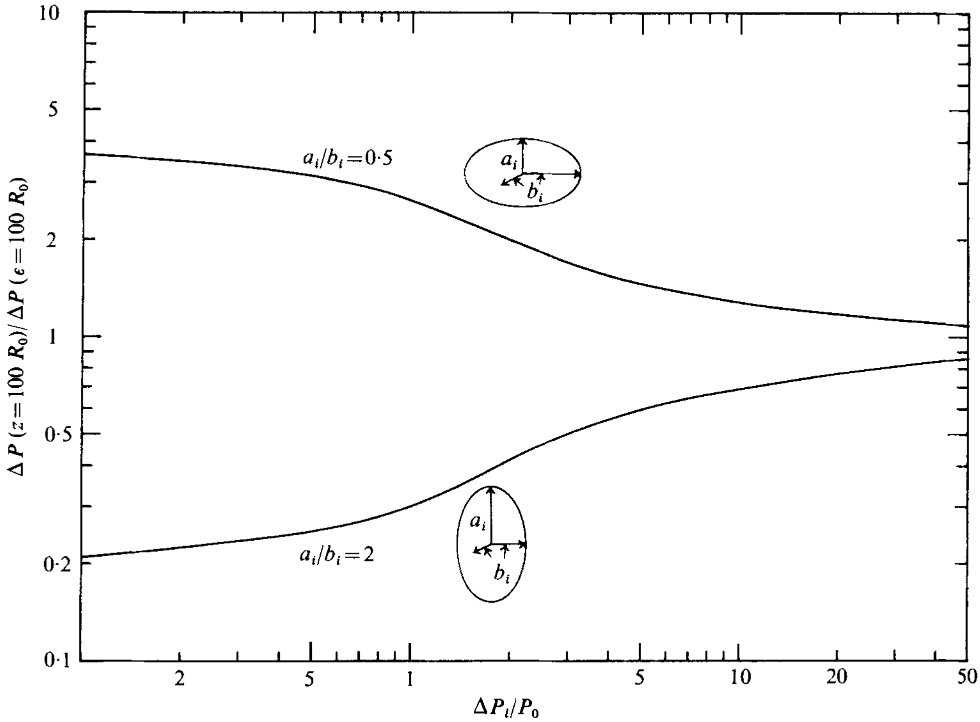


FIGURE 4. A plot of the ratio of the overpressure along the major axis to that along the minor axis in the far field (at 100 explosion lengths) as function of the initial overpressure.

R_0 is the explosion length. For each case, the corresponding curve for the spherical blast of the same total energy and overpressure ratio is shown for comparison. For the case $\Delta P_i/P_0 = 0{\cdot}1$, of a weak initial overpressure ratio, the limiting solutions from the acoustic theory of §4 are also plotted.

In figure 3(*a*), where $a_i/b_i = 2$, we note that the decay of the blast wave along the major axis is much more rapid than the decay for the corresponding spherical case or the decay along the minor axis in the central x, y plane. This is due to the large curvature of the shock wave along the z axis. As a result of the slower decay rate, the blast overpressure along the minor axis quickly becomes greater than that along the major axis. This degree of directionality is more severe for initially weaker blast waves than for stronger shocks.

Similar results obtained for the case $a_i/b_i = 0{\cdot}5$ are shown in figure 3(*b*). Here the blast decays much more rapidly along the minor axis than along the major axis since the curvature along the minor axis (the x, y plane) is greater than that along the major axis. Again the degree of directionality is less for stronger shocks, and the shock decay approaches the spherical case in the far field when $a/b \rightarrow 1$. In both figure 3(*a*) and figure 3(*b*), we note that the results for the peak overpressure from the B–K theory agree extremely well with those from the acoustic theory when the initial overpressure is small and the blast wave generated is weak.

In figure 4 we show quantitatively the ratio of the overpressure along the major axis (z axis) to that along the minor axis (x, y plane) in the far field (at one hundred explosive lengths) as function of the initial overpressure. In figure 5 the peak over-

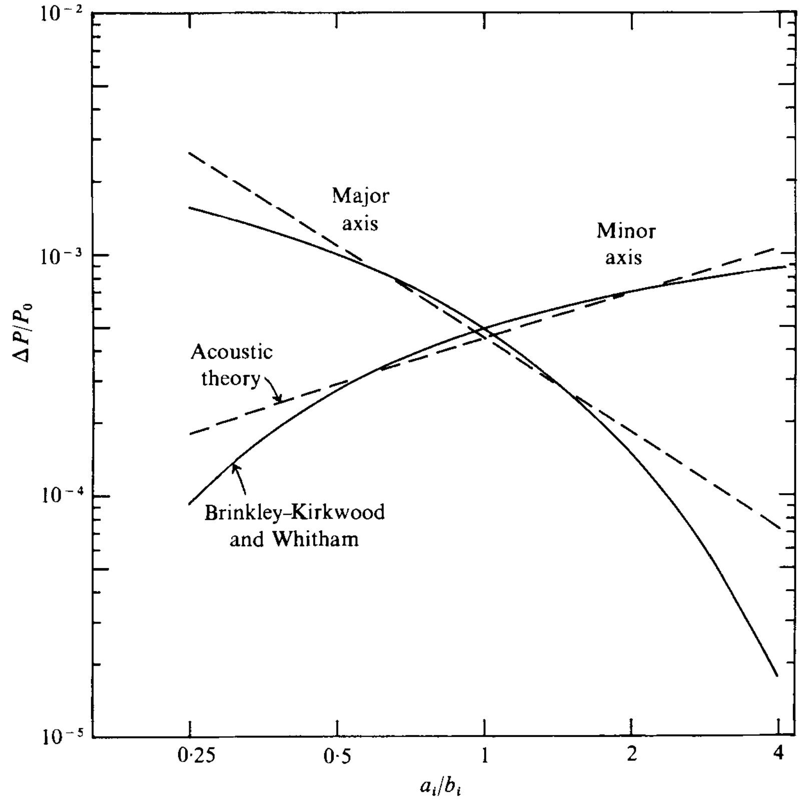


FIGURE 5. The peak overpressure along the major and the minor axis at $r = 100R_0$ plotted against a_i/b_i, the degree of ellipticity of the exploding ellipsoid.

pressures along the major and the minor axes at $r = 100R_0$ are plotted against a_i/b_i, the degree of ellipticity of the exploding ellipsoidal vessel. The solid curves are results of the Brinkley–Kirkwood theory and Whitham's ray-shock theory. The dashed curves were obtained by using the modified acoustic theory. Agreement is very good for $0{\cdot}5 < a_i/b_i < 2$. We thus conclude that the present theory with the assumption that the shock surface is ellipsoidal at all times is good only in the range $0{\cdot}5 < a_i/b_i < 2$. For highly asymmetric situations, the shock shape will not remain ellipsoidal all the time. Such problems can be solved only numerically.

An experiment was performed to verify the present theory. The asymmetrical blast wave was generated by exploding a 2 cm long fine copper wire with a high voltage capacitor energy source. The shock shapes at various times were recorded via high-speed framing schlieren photography. A typical result is shown in figure 6. From the schlieren photographs, the shock trajectories along the major axis and the minor axis were obtained. As can be observed, the shock shapes are not ellipsoidal. However, we assumed them to be ellipsoidal and used the present theory to obtain the shock trajectories for comparison with the experimental results. In the experimental situation, the blast is not generated by the rupture of a pressurized ellipsoid. Hence we used a different set of initial conditions to start the numerical integration of (3.7)–(3.12). From the experimental shock trajectories, we obtained the shock strength and its rate of decay. Thus, choosing an arbitrary time as the initial instant at which the

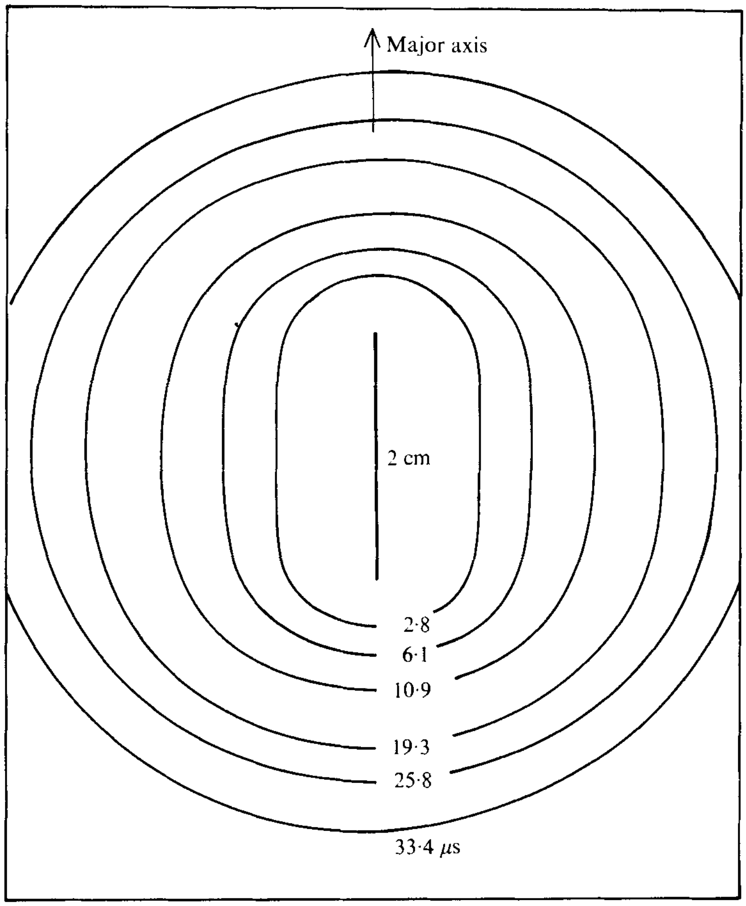


FIGURE 6. The shock shapes at various times after the explosion of a 2 cm long copper wire as recorded by schlieren photography.

shock shape, Mach number and rate of shock decay were obtained from experiments, (3.7)–(3.12) were integrated numerically for the subsequent motion of the blast. A comparison of the shock trajectories along the major and minor axes is given in figure 7. It is to be noted that the agreement is quite good in spite of the assumption that the shock shape is ellipsoidal at all times, which is not the case experimentally. Thus it appears that this assumption is not too critical, and it is felt that an asymmetrical blast of arbitrary rotational symmetry can often be described quite well by the simple theory developed in this paper.

6. Conclusions

The present theory for the propagation of asymmetric blast waves is based on the Brinkley–Kirkwood theory and Whitham's ray-shock theory. Although it is an approximate theory, the task involved in obtaining a solution for the motion of an arbitrarily shaped blast wave is by no means trivial. However, if the shock shapes at all times can be specified, then the dynamics of the asymmetrical blast can readily be determined from the numerical integration of a pair of ordinary differential equations.

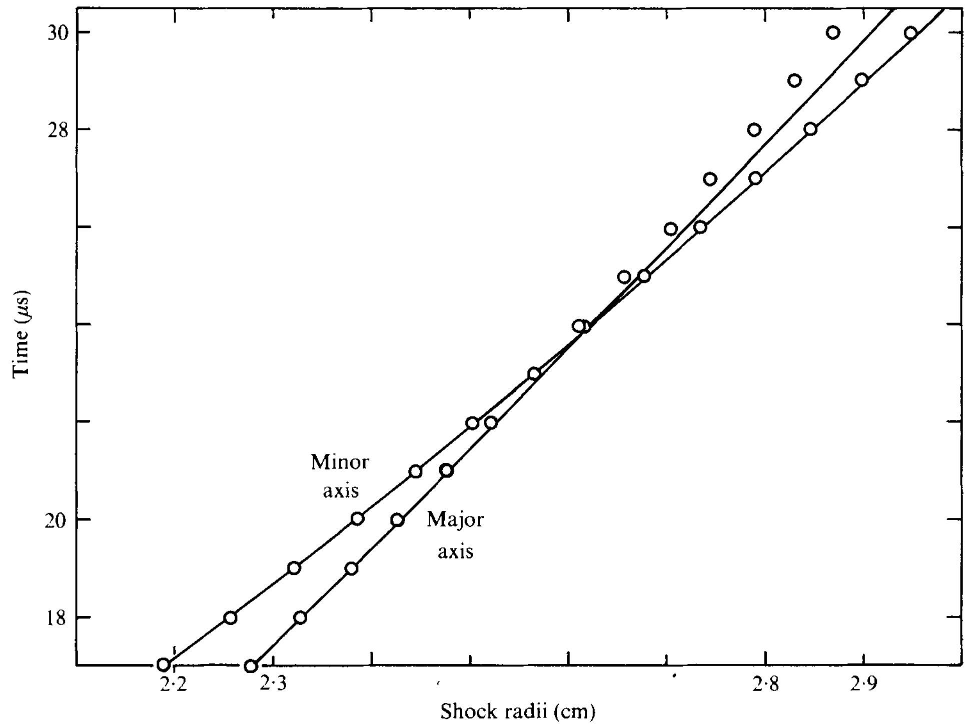


FIGURE 7. A comparison of the present theory with experimental results obtained from figure 6. ——, theory; ○, experiment.

It should be noted that the simplification results from the decoupling of the geometrical from the dynamical relationships, and this is achieved once the shock shape or how it changes with time is known, or assumed. As a matter of fact, we do not require in this paper that the shock shapes be similar to achieve this decoupling. We merely demand that the shock shape remains ellipsoidal for all times, and the degree of ellipticity may change with time. Thus this assumption is not too restrictive and is found to be extremely good when the degree of ellipticity is within the range $0{\cdot}5 < a_i/b_i < 2$.

Of particular importance in the present study is the demonstration that, for the same degree of initial asymmetry, a weaker blast has a more severe degree of directionality in the far field than a stronger blast. Thus in accidental explosions, where the blast waves are in general weak, asymmetry is an important factor and should be taken into consideration in the assessment of blast damage and risk evaluation.

A modified acoustic theory is also given for an elliptical explosion with nonlinear wave distortion taken into account. For weak shock waves, this acoustic theory agrees extremely well with the more general theory developed from the Brinkley–Kirkwood theory. Experimental results for asymmetrical blast waves generated by exploding wires are found to agree quite well with the present theory.

We have also investigated the static and the dynamic impulses of the asymmetrical blast wave by making use of the original assumptions for the pressure–time integrals as given in the paper of Brinkley & Kirkwood. Unfortunately, the results do not approach the proper limits for weak shocks as predicted by the acoustic theory. In view of all the simplifying assumptions involved, we conclude that the Brinkley–Kirkwood theory, though it gives excellent results for the peak overpressure and the shock trajectories, fails to yield a quantitative description of the flow profile. Further improvement is necessary.

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